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TOWARDS NUMERICAL AND ANALYTICAL STUDIES OF FIRST ORDER PHASE TRANSITIONS [a]

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Discrete lattice simulations of a one-dimensional ϕ^4 theory coupled to an external heat bath are being carried out. Great care is taken to remove the effects of lattice discreteness and finite size and to establish the correct correspondence between simulations and the desired, finite-temperature continuum limit.

1 Introduction

To be able to study numerically certain properties of cosmological first order phase transitions such as the nucleation rate of bubbles, we investigate first the effects that finiteness and discreteness of a lattice have on results derived from simulations. For this basic study we limit ourselves to one spatial dimension and to a real scalar field $\phi(x,t)$ subjected to a potential $V_0(\phi)$ and an environmental temperature T. The dynamics obey a Langevin equation as given in Borrill and Gleiser [1]. This 2D-paper indicates that for a given tree-level potential, results obtained numerically on the lattice can't straightforwardly be identified with analytical results. They employ a renormalization procedure to get rid of the lattice spacing dependence and to identify the correct continuum limit of the simulations. Similar problems in 1D are treated in what follows.

2 Method

For classical field theories, the one-loop corrected effective potential is given by a momentum integral [2], and evaluated to

$$V_{1\mathrm{L}}(\phi) = V_0(\phi) + \frac{T}{2}\int_0^\infty \frac{dk}{2\pi}\ln\left[1+\frac{V_0''(\phi)}{k^2}\right] = V_0(\phi) + \frac{T}{4}\sqrt{V_0''(\phi)}\;. \qquad (1)$$

The discretization δx of the lattice and its finite size L introduce short and long momentum cutoffs $k_{\min} = 2\pi/L$ and $\Lambda = \pi/\delta x$. Therefore the simulation only sees $\tilde{V}_{1\mathrm{L}} = V_0(\phi) + (T/2)\int_{k_{\min}}^{\Lambda}\ldots dk$. If one neglects the effect of $k_{\min}$

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which is possible for sufficiently large L [b], integration and expansion in powers of V_0''/Λ^2 (possible for sufficiently large Λ) yields

$$\tilde{V}_{1\mathrm{L}} = V_0 + \frac{T}{4}\sqrt{V_0''} - \frac{T}{4\pi}\frac{V_0''}{\Lambda} + \Lambda\, O\left(\frac{V_0''^2}{\Lambda^4}\right) \ . \tag{2}$$

As is to be expected for a one dimensional system, the limit $\Lambda \to \infty$ exists and is well-behaved; there is no need for renormalization due to divergences. However, the effective one-loop potential is lattice-spacing dependent through the explicit appearance of Λ, and so are the corresponding numerical simulations, as evidenced in the tree-level potential cases of Fig. 1 (left graphs).

Similar to the renormalization procedure for 2D systems given by Borrill and Gleiser [1] we remove this dependence on δx by adding counterterms to the tree-level potential V_0. In contrast to higher-dimensional systems, these counterterms are *finite*, namely $V_{\mathrm{CT}}(\phi) = (T/4\pi)(V_0''(\phi)/\Lambda)$. Hence the lattice simulation works with the corrected potential

$$V_{\mathrm{Latt}}(\phi) = V_0(\phi) + \frac{TV_0''(\phi)}{4\pi^2}\delta x \tag{3}$$

and simulates the continuum limit to one loop, $\tilde{V}_{1\mathrm{L}}(\phi) = V_0 + (T/2)\int_0^\Lambda \ldots dk = V_{1\mathrm{L}}(\phi)$ (where V''_{Latt} is employed in the integrand), just as it should be.

3 Application

Since the numerical extraction of bubble nucleation rates is a contrived process the ideas of Section 2 are tested initially with the *symmetric* double well potential $V_0(\phi) = (\lambda/4)\left(\phi^2 - \phi_0^2\right)^2$. We compare simulations using V_0 alone with those employing $V_{\mathrm{Latt}}(\phi) = V_0(\phi) + 3\lambda T\delta x\phi^2/4\pi^2$ (eq. 3). One set of runs investigates the mean field value $\langle\bar{\phi}\rangle$ of the metastable equilibrium before the first kink-antikink pair occurs ($\bar{\phi}(t) = (1/L)\int \phi(x,t)dx$). Another set of runs measures the kink-antikink pair density n_{p} (proportional to the number of zeros of the low-pass filtered field). Fig. 1 shows the comparison for different lattice spacings δx. Apart from a discrepancy for very coarse grids ($\delta x \approx 1$) the average field value is clearly lattice-spacing independent in the right panel (V_{Latt}), in contrast to the use of V_0. The effect is even more striking in the case of n_{p} where the addition of the finite counterterm removes any δx dependence.

[b] Lattice simulations only know one size parameter, the number of degrees of freedom $N = L/\delta x$. With a given N it is always possible to choose L big enough for the effects of δx to dominate over those of L.

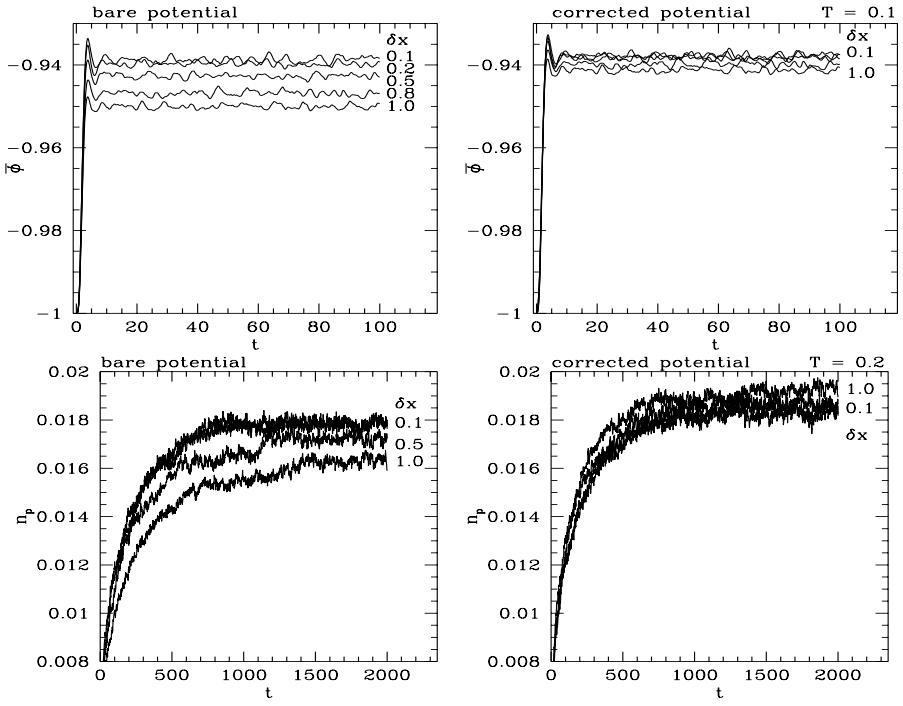

Figure 1: Average field value $\bar{\phi}(t)$, top, and density of kink-antikinks (half of density of zeros), bottom, using the tree-level potential, left, and the corrected potential, right.

In summary it was demonstrated that even in a field theory without divergences finite counterterms play a role. Their inclusion gets rid of the dependence of simulations on size and lattice spacing. This can be observed in the averaged field value and in the density of kink-antikinks. Further studies of this renormalization procedure [3] identify the correct continuum limit of simulations, thus matching theory and numerical results.

Acknowledgments

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